Session 2
Part 1
Cost of Equity and Portfolio Analysis; Cost of Debt and Bond Valuation

Financial Markets and Management

MiM

ISEG Lisbon School of Economics & Management

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Bibliographic References

- In any of the standard Corporate Finance textbooks, the chapters on:
  - Stock/Equity Valuation and Portfolios
  - Bond Valuation.

Outline

Cost of Capital:

Cost of Equity & Cost of Debt

- We will study the capital structure choice of firms – i.e., their composition of equity and debt financing.
- So we start by examining simple models to calculate the costs of financing of the two sources of capital.

1. Cost of Equity:
   - Based on the **simplest models of dividend growth**;
   - Based on **portfolio analysis and the CAPM** (Capital Asset Pricing Model).
2. Cost of Debt:
   - Based on the **simplest bond valuations** and their yields to maturity;
   - Based on the **CAPM**.

- Later, we will study how the capital structure choice of a firm might influence the overall cost of capital of the firm.
- Finally we will have a look at the practice of capital structure decisions: how firms do finance themselves, what are the costs involved in new issues, etc.

1. Cost of Equity (r_E):
1.1 The Dividend Discount Model

- Start with a one-year investor (buy now, sell after 1 year).
  - The timeline with associated cash flows would be:

0	1
$-P_0$	$Div_1 + P_1$

  - Since the cash flows are risky, we must discount them at the **equity cost of capital (r_E).** Price would be:
$$\boxed{P_0 = \left(\frac{Div_1 + P_1}{1 + r_E}\right)}$$
  - Total Equity Return is due to:
$$\boxed{r_E = \frac{Div_1 + P_1}{P_0} - 1 = \underbrace{\frac{Div_1}{P_0}}_{\text{Dividend Yield}} + \underbrace{\frac{P_1 - P_0}{P_0}}_{\text{Capital Gain Rate}}}$$

The Dividend Discount Model (cont.)

- Example:
  - 3M (MMM) is expected to pay dividends of \$1.92 per share in the coming year.
  - You expect the stock price to be \$85 per share at the end of the year.
  - Investments with equivalent risk have an expected return of 11%.
    - What is the most you would pay today for 3M stock?
    - What dividend yield and capital gain rate would you expect at this price?

$$P_0 = \frac{Div_1 + P_1}{(1 + r_E)} = \frac{\$1.92 + \$85}{(1.11)} = \$78.31$$

$$\text{Dividend Yield} = \frac{Div_1}{P_0} = \frac{\$1.92}{\$78.31} = 2.45\%$$

$$\text{Capital Gains Yield} = \frac{P_1 - P_0}{P_0} = \frac{\$85.00 - \$78.31}{\$78.31} = 8.54\%$$

- Total Return = 2.45% + 8.54% = 10.99% ≈ 11%

The Dividend Discount Model: with Constant Growth Rate

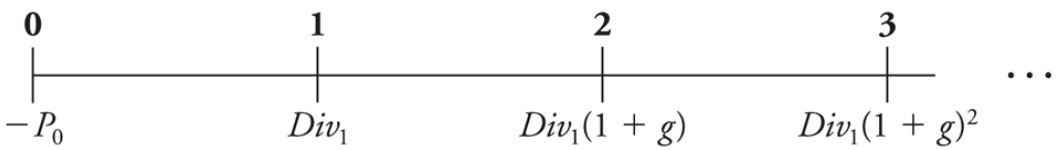
- What is the price if we plan on holding the stock for N years?

$$P_0 = \frac{Div_1}{1 + r_E} + \frac{Div_2}{(1 + r_E)^2} + \cdots + \frac{Div_N}{(1 + r_E)^N} + \frac{P_N}{(1 + r_E)^N}$$

  - This is known as the Dividend Discount Model. Therefore:

$$P_0 = \frac{Div_1}{1 + r_E} + \frac{Div_2}{(1 + r_E)^2} + \frac{Div_3}{(1 + r_E)^3} + \cdots = \sum_{n=1}^{\infty} \frac{Div_n}{(1 + r_E)^n}$$

- How to Apply the Dividend Discount Model?
  - One possibility is assuming **Constant Dividend Growth**, at a constant rate, g, forever.



The Dividend Discount Model: with Constant Growth Rate (cont.)

- With the Constant Dividend Growth Model we have:

$$\boxed{P_0 = \frac{Div_1}{r_E - g}} \qquad \boxed{r_E = \frac{Div_1}{P_0} + g}$$

- Example:
  - AT&T plans to pay $1.44 per share in dividends in the coming year.
  - Its equity cost of capital is 8%.
  - Dividends are expected to grow by 4% per year in the future.
  - **Estimate the value of AT&T's stock.**

$$P_0 = \frac{Div_1}{r_E - g} = \frac{\$1.44}{.08 - .04} = \$36.00$$

The Dividend Discount Model: with Constant Growth Rate (cont.)

- Where does the growth rate g come from? A simple model assumes:

$$\boxed{r_E = \frac{Div_1}{P_0} + g}$$

$$Div_t = \underbrace{\frac{\text{Earnings}_t}{\text{Shares Outstanding}_t}}_{EPS_t} \times \text{Dividend Payout Rate}_t$$

Dividend Payout Rate: the percentage of earnings distributed as dividends.

$$g = \text{Retention Rate} \times \text{Return on New Investment}$$

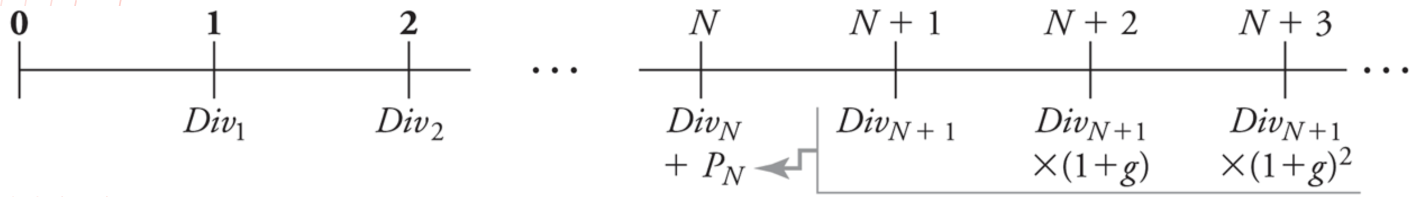
Retention Rate: the fraction of earnings that the firm reinvests

- Note: if a firm keeps its Retention Rate constant, the growth rate in dividends will equal the growth rate in earnings.

The Dividend Discount Model: With Changing Growth rates

We cannot use the constant dividend growth model to value a stock if the growth rate is not constant.

But we can use the general form of the model to value a firm by applying the constant growth model to calculate the future share price of the stock once the expected growth rate stabilizes. Timeline would be:



With constant growth from year N+1 onwards: $\boxed{P_N = \dfrac{Div_{N+1}}{r_E - g}}$

The Dividend Discount Model: With Changing Growth rates (cont.)

- Finally, the Dividend-Discount Model with Constant Long-Term Growth gives us:

$$\boxed{P_0 = \frac{Div_1}{1 + r_E} + \frac{Div_2}{(1 + r_E)^2} + \cdots + \frac{Div_N}{(1 + r_E)^N} + \frac{1}{(1 + r_E)^N}\left(\frac{Div_{N+1}}{r_E - g}\right)}$$

The Total Payout Model

- With a Stock Repurchase the firm uses excess cash to buy back its own stock.
- If a firm uses Stock Repurchases as a method of paying out, the DDM is restrictive:
  - The more cash the firm uses to repurchase shares, the less it has available to pay dividends.
  - By repurchasing, the firm decreases the number of shares outstanding, which increases its earnings per and dividends per share.
- With Share Repurchases we may use the Total Payout Model.

The Total Payout Model (cont.)

- The Total Payout Model starts by valuing total equity of the firm.
  - You discount total dividends and share repurchases and use the growth rate of earnings (rather than earnings per share) when forecasting the growth of the firm’s total payouts.
  - Lastly you divide total equity by the number of shares outstanding.

$$PV_0 = \frac{PV(\text{Future Total Dividends and Repurchases})}{\text{Shares Outstanding}_0}$$

The Total Payout Model (cont.)

- Example:
  - Titan Industries has 217 million shares outstanding, and expects earnings of $860 million at the end of this year. The equity cost of capital is 10%.
  - Titan plans to pay out 50% of its earnings, paying 30% as a dividend and using 20% to repurchase shares. The payout rates are expected to remain constant.
  - Titan's earnings are expected to grow by 7.5% per year.

- What's the expected share price of Titan Industries?
  - Total Payout Year 1= 50%x$860 million = $430 million
  - PV(Future Total Dividends and Repurchases)=
  - Price per share = Po = 17.2 billion/217 million shares = $79.26

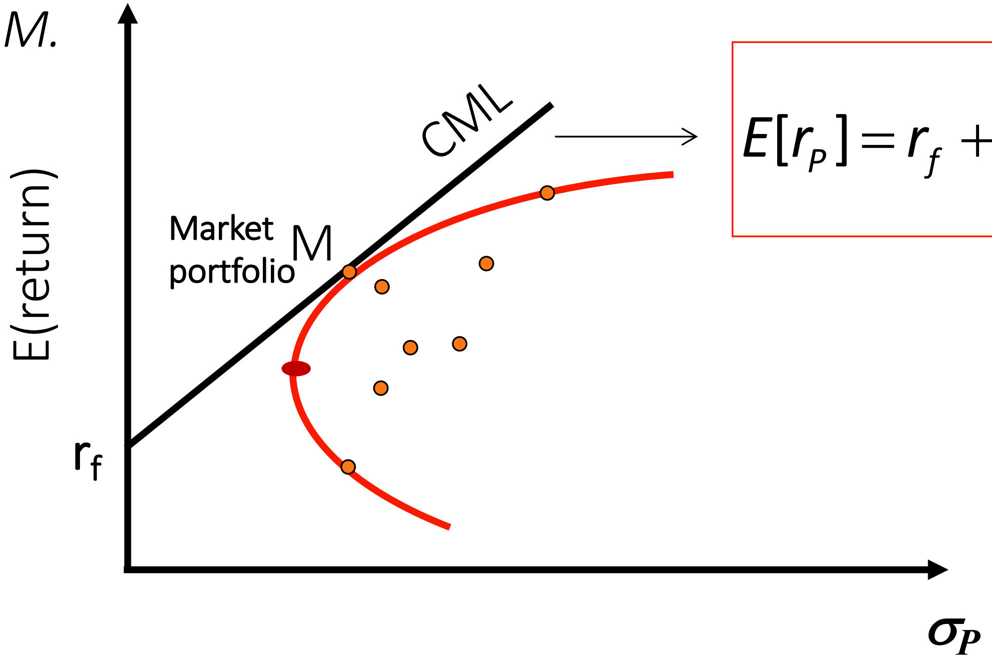
1. Cost of Equity (r_E):
1.2 Capital Asset Pricing Model

- The Capital Asset Pricing Model (CAPM) is an equilibrium model that establishes a relationship between the price of a security and its risk.
- In particular, the CAPM is used to determine the cost of capital: minimum return required by investors for a certain level of risk.
- The CAPM assumes investors are well diversified. Hence the risk premium is proportional to a measure of market (or systematic) risk, known as Beta.

Remember Portfolio Theory?
Capital Market Line

Investors choose a point along the line – Capital Market Line (CML).

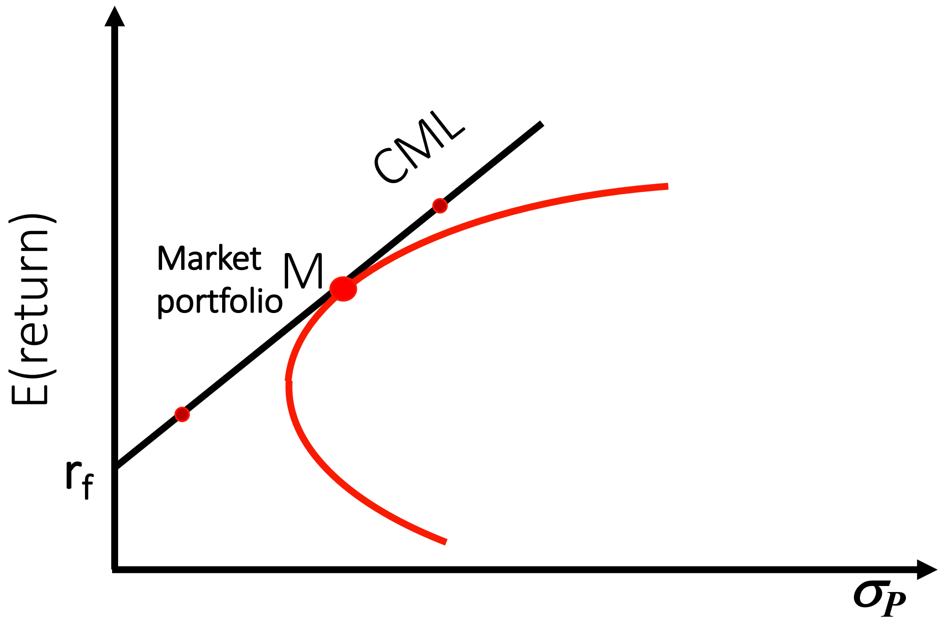
Efficient portfolios are combination of the risk-free asset and the market portfolio *M.*



$$E[r_P] = r_f + \left(\frac{E[r_M] - r_f}{\sigma_M} \right) \sigma_P$$

Capital Market Line

- Where the investor chooses to be along the CML depends on his risk aversion – but all investors face the same CML.



Security Market Line

- The CML gives risk-return trade-off for efficient portfolios.
- In equilibrium, what is the relation between expected return and risk for individual stocks?
  - Individual stocks are below CML.
  - This relation is named Security Market Line (SML).
  - Individual stock risk is measured by its covariance with the market portfolio because it is the *marginal variance.*
    - How does a small increment to the weight of a stock change the variance of the portfolio?
    - As in Economics, it is the marginal cost of goods that determines their prices, not their total or average cost.

Beta

- According to the Security Market Line, for any security *i*:

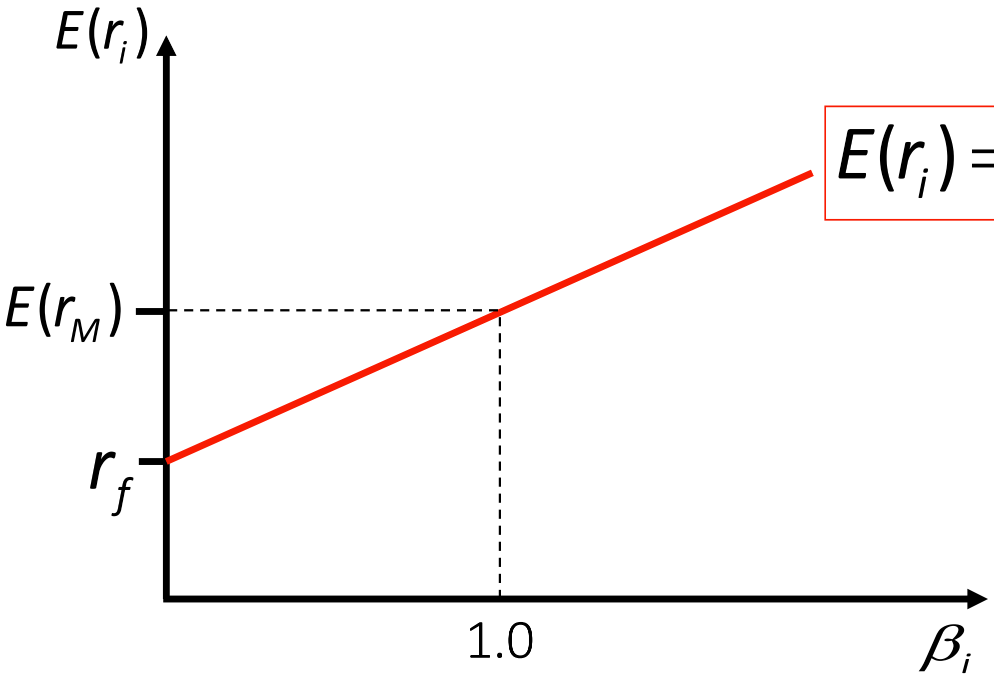
$$\boxed{E(r_i) = r_f + \beta_i \, [E(r_M) - r_f]}$$

- where

$$\boxed{\beta_i = \frac{Cov(r_i, r_M)}{Var(r_M)}}$$

- Beta measures the responsiveness of a stock to movements in the market portfolio (i.e., systematic risk).

CAPM: Expected Return of a Stock



$$E(r_i) = r_f + \beta_i [E(r_M) - r_f]$$

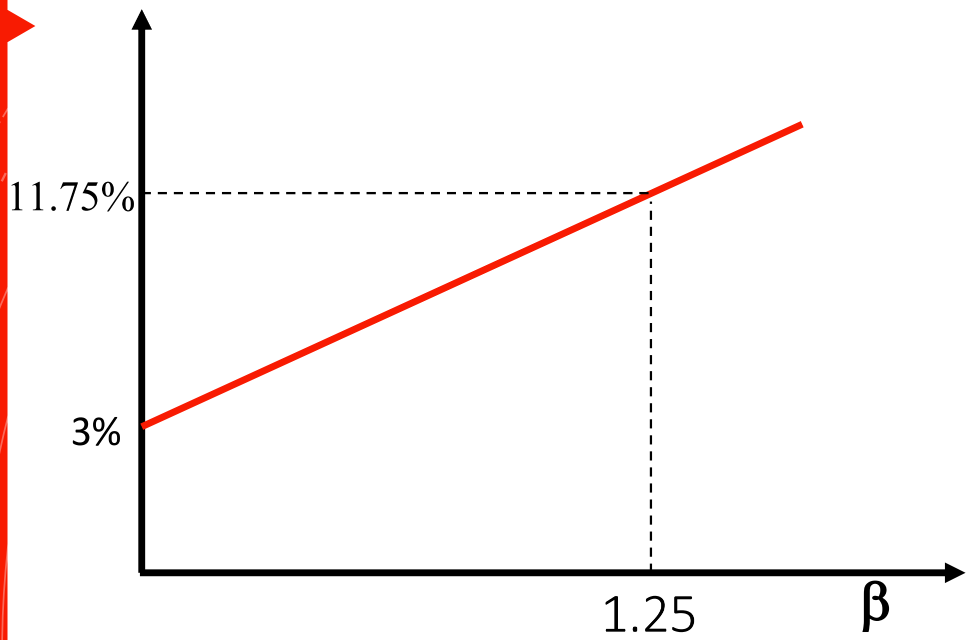
CAPM: Expected Return of a Stock

$$E(r_i) = r_f + \beta_i [E(r_M) - r_f]$$

Expected return on a stock	=	Risk-free rate	+	Beta of stock	×	Market risk premium

- Assume $\beta_i = 0$, then $E(r_i) = r_f$
- Assume $\beta_i = 1$, then $E(r_i) = E(r_M)$
- Assume $\beta_i < 1$, then $E(r_i) < E(r_M)$
- Assume $\beta_i > 1$, then $E(r_i) > E(r_M)$

CAPM: Example



$$\beta_i = 1.25 \quad r_f = 3\% \quad E(r_M) = 10\%$$

$$E(r_i) = 3\% + 1.25 \times (10\% - 3\%) = 11.75\%$$

Beta of a Portfolio

- Beta of a portfolio is the portfolio-weighted average of individual assets:

$$\boxed{\beta_P = \sum_{i=1}^{N} w_i \beta_i}$$

- Thus, we can use SML for any portfolio:

$$\boxed{E(r_P) = r_f + \beta_P [E(r_M) - r_f]}$$

Why Beta?

- Because investors can diversify their portfolios, they only require a risk premium for non-diversifiable (market, systematic) risk. This is what **Beta** measures.
- High beta stocks are risky, and must therefore offer a higher return on average to compensate for the risk
- Why are high beta stocks risky?
  - Because they pay up just when you need the money least, when the overall market is doing well
  - And they lose money when you really need it – when the overall market is doing poorly
  - If anyone is to hold this security, it must offer a high expected return

Estimation of Beta

- β_i usually estimated using a time-series regression

$$r_{i,t} - r_{f,t} = \alpha_i + \beta_i (r_{M,t} - r_{f,t}) + \varepsilon_{i,t}$$

- Typical R^2=25%
- Estimation issues
  - Betas may change over time
  - Data might be too old
  - Five years of weekly or monthly data is reasonable
  - Use Data Analysis / Regression or Linest in Excel

Variables to use for the Market Return and the Risk-free rate

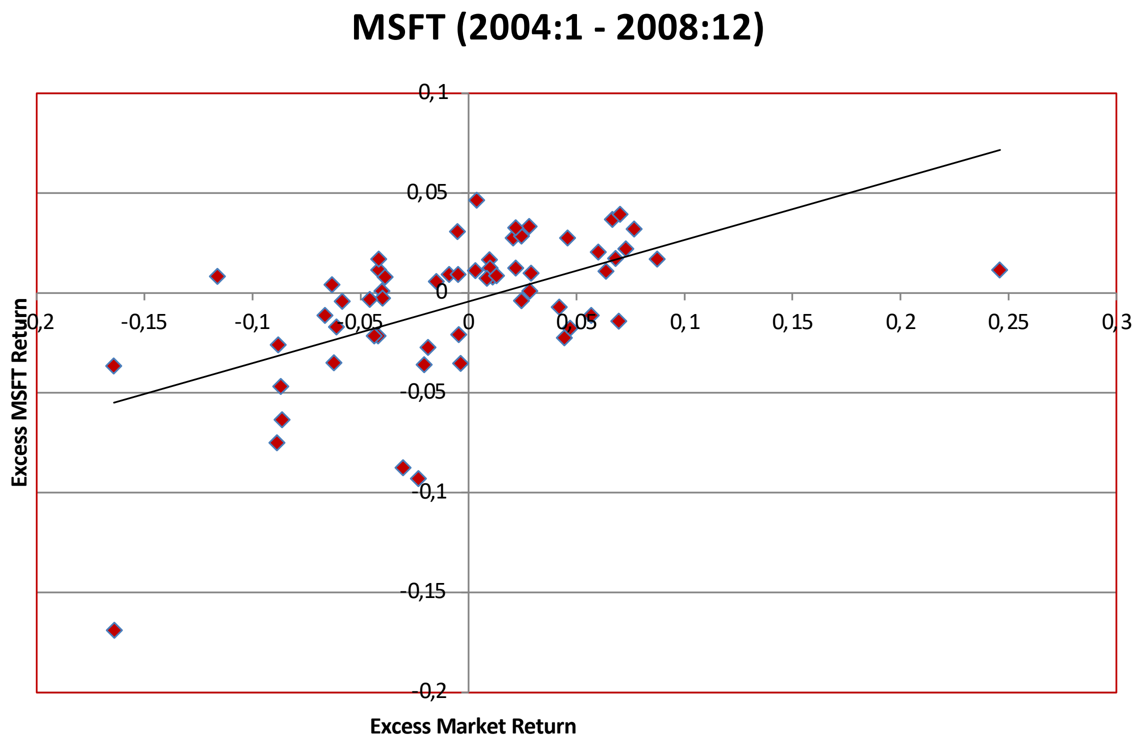
- What market proxy?
  - CAPM says it should be all the assets in the world
  - Typically people use *broad, value-weighted* stock market index (e.g. S&P 500)
- What risk-free rate?
  - CAPM says it should be riskless and match the horizon of the application
  - People use short-term sovereign debt: T-bills

Market Risk Premium

- This is the hardest input to measure in the CAPM equation
- From January 1926 to December 2005, the excess market return has been 6.7%
  - Depending on the sample and on whether we use the arithmetic or geometric mean, we can come up with numbers between 5% and 8%
- Can we trust this historical average?
  - Standard error of the estimate is 2.2%

Example: Estimating MSFT's Beta

$$r_{MSFT,t} - r_{f,t} = 0.002 + 0.993(r_{M,t} - r_{f,t}) + \varepsilon_{MSFT,t}$$



Example: Estimating MSFT's stock Expected Return

- Assuming a risk-free rate of 3% and an equity premium of 6%, the expected return (annual) on MSFT shares would be:

$$r_E^{Microsoft} = 3\% + 0.993 \times 6\% = 8.96\%$$

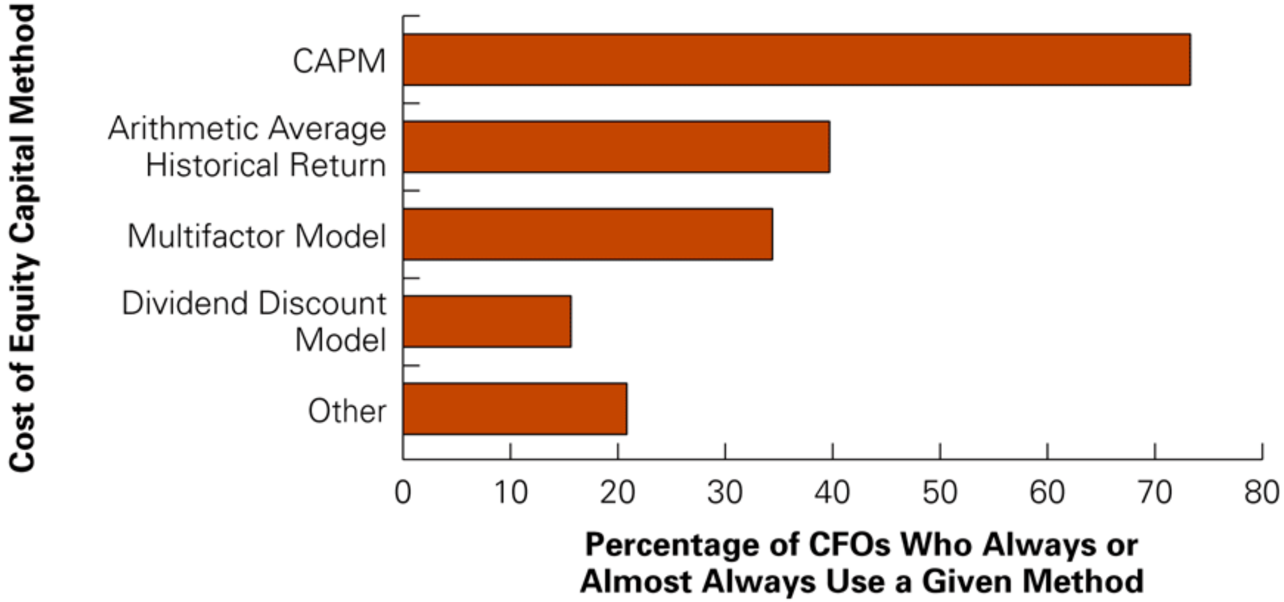
- Another Example:
  - Suppose you estimate that Google's stock has a volatility of 26% and a beta of 1.45.
  - The risk-free interest rate is 3% and you estimate the market's expected return to be 8%.
  - Google's cost of equity capital would be:

$$r_E^{Google} = 3\% + 1.45 \times (8\% - 3\%) = 10.25\%$$

Special Concerns with the CAPM

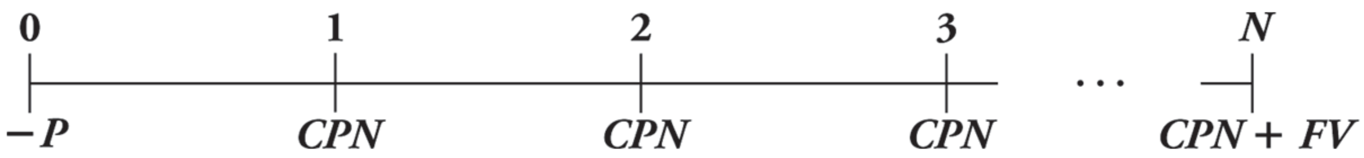
- Betas of shares vary with the debt levels of firms:
  - when leverage changes, beta of equity changes.
- Betas of projects can differ from betas of firms:
  - especially in well-diversified firms.
- When firms are not listed in a stock exchange, how to use the CAPM?
  - Many times by finding Comparable firms in the same industry that are publicly listed, and estimating an industry beta.

The Equity Cost of Capital (r_E)



Source: J. R. Graham and C. R. Harvey, "The Theory and Practice of Corporate Finance: Evidence from the Field," Journal of Financial Economics 60 (2001): 187–243.

- A Coupon Bond:
  - Pays face value at maturity
  - Pays regular coupon interest payments



- Yield to Maturity: The YTM is the *single* discount rate that equates the present **value of the bond's remaining cash flows to its current price.**

2. Cost of Debt: Think of a Simple Bond

Think of a Simple Coupon-Paying Bond

- Yield to Maturity of a Coupon-paying Bond:

$$P = CPN \times \frac{1}{y}\left(1 - \frac{1}{(1+y)^N}\right) + \frac{FV}{(1+y)^N}$$

- Example: Consider the following annual coupon bond:
  - \$1000 par value
  - 14 years until maturity
  - 4.5% coupon rate
  - Price is \$1,080.55
- What is the bond's yield to maturity?

$$\$1,080.55 = (0.045 \times \$1,000) * \frac{1}{y}\left[1 - \frac{1}{(1+y)^{14}}\right] + \frac{\$1,000}{(1+y)^{14}}$$

y =3.75%

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The Debt Cost of Capital (r_D)

- The most common way of estimating the Cost of Debt is using Debt Yields:
  - Yield to maturity is the IRR an investor will earn from holding the bond to maturity and receiving its promised payments.

  - *If there is significant risk of default, yield to maturity will overstate investors' expected return.*
    - In that case we must adjust for the *truly expected return* for the firm's creditors, taking into account the probability of default and the amount of expected loss in case of default.

The Debt Cost of Capital (r_D)

- Consider a one-year bond with YTM of *y*. For each \$1 invested in the bond today, the issuer promises to pay \$(1+*y*) in one year.
- Suppose the bond will default with probability *p*, in which case bond holders receive only \$(1+*y*-*L*), where *L* is the expected loss per \$1 of debt in the event of default.
- So the expected return of the bond is:

$$r_D = (1-p)y + p(y-L) = y - pL =$$

$$= \text{Yield to Maturity} - \text{Prob(default)} \times \text{Expected Loss Rate}$$

The Debt Cost of Capital (r_D)

- Annual Default Rates by Debt Rating (1983–2008):

Rating:	AAA	AA	A	BBB	BB	B	CCC	CC-C
Default Rate:								
Average	0.0%	0.0%	0.2%	0.4%	2.1%	5.2%	9.9%	12.9%
In Recessions	0.0%	1.0%	3.0%	3.0%	8.0%	16.0%	43.0%	79.0%

Source: "Corporate Defaults and Recovery Rates, 1920–2008," *Moody's Global Credit Policy*, February 2009.

- The average loss rate for unsecured debt is 60%.

- Example: The expected return to B-rated bondholders during average times is 0.052X0.60=3.1% below the bond's quoted yield.

The Debt Cost of Capital (r_D)

- Another way of estimating the Cost of Debt would be using the CAPM and Debt Betas.
  - Debt betas are difficult to estimate because corporate bonds are traded infrequently.
  - One approximation is to use estimates of betas of bond indices by rating category.

By Rating	*A and above*	*BBB*	*BB*	*B*	*CCC*
Avg. Beta	< 0.05	0.10	0.17	0.26	0.31
By Maturity	(BBB and above)	*1–5 Year*	*5–10 Year*	*10–15 Year*	*> 15 Year*
Avg. Beta		0.01	0.06	0.07	0.14

Source: S. Schaefer and I. Strebulaev, "Risk in Capital Structure Arbitrage," Stanford GSB working paper, 2009.

The Debt Cost of Capital (r_D)

- Example:
  - In mid-2009, homebuilder KB Home had outstanding 6-year bonds with a YTM of 8.5% and a BB rating.
  - Consider that the corresponding risk-free rate was 3%, and the market risk premium is 5%.
  - KB Home's Cost of Debt was:
    - Using the YTM, the probability of default of BB rating bonds, and the expected loss in default of 60%: $r_D^{KBHome} = 8.5\% - 8\% \times 0.6 = 3.7\%$
    - Using the Debt Beta of a BB rating bond, and CAPM:

$$r_D^{KBHome} = 3\% + 0.17 \times 5\% = 3.85\%$$

The Cost of Capital of a Firm

- It is common to compute the weighted average of the costs of equity and debt. This rate is known as the WACC (weighted average cost of capital).
- (In the absence of corporate taxes) the pre-tax WACC is computed as:

$$r_{wacc} = \frac{E}{E+D} r_E + \frac{D}{E+D} r_D$$

- In the presence of corporate taxes, due to the more favorable tax treatment given to debt financing, the WACC is computed as:

$$\boxed{r_{wacc} = \frac{E}{E+D} r_E + \frac{D}{E+D} r_D (1 - T_C)}$$

  - Note: The value of Debt considered here can be understood as "Net Debt" – by this we mean debt net of excess cash that the firm might hold. The denominator is the Enterprise Value.

Appendix 1: Bond Ratings

- Several rating agencies (Moody’s S&Ps, Fitch) classify bond issues of firms according to their risks.
- They make a clear distinction between
  - Investment Grade Bonds, and
  - Speculative Bonds
    - Also known as Junk Bonds or High-Yield Bonds

Appendix 1: Bond Ratings (cont.)

Rating*	Description (Moody's)
Investment Grade Debt	
Aaa/AAA	Judged to be of the best quality. They carry the smallest degree of investment risk and are generally referred to as "gilt edged." Interest payments are protected by a large or an exceptionally stable margin and principal is secure. While the various protective elements are likely to change, such changes as can be visualized are most unlikely to impair the fundamentally strong position of such issues.
Aa/AA	Judged to be of high quality by all standards. Together with the Aaa group, they constitute what are generally known as high-grade bonds. They are rated lower than the best bonds because margins of protection may not be as large as in Aaa securities or fluctuation of protective elements may be of greater amplitude or there may be other elements present that make the long-term risk appear somewhat larger than the Aaa securities.
A/A	Possess many favorable investment attributes and are considered as upper-medium-grade obligations. Factors giving security to principal and interest are considered adequate, but elements may be present that suggest a susceptibility to impairment some time in the future.
Baa/BBB	Are considered as medium-grade obligations (i.e., they are neither highly protected nor poorly secured). Interest payments and principal security appear adequate for the present but certain protective elements may be lacking or may be characteristically unreliable over any great length of time. Such bonds lack outstanding investment characteristics and, in fact, have speculative characteristics as well.
Speculative Bonds	
Ba/BB	Judged to have speculative elements; their future cannot be considered as well assured. Often the protection of interest and principal payments may be very moderate, and thereby not well safeguarded during both good and bad times over the future. Uncertainty of position characterizes bonds in this class.
B/B	Generally lack characteristics of the desirable investment. Assurance of interest and principal payments of maintenance of other terms of the contract over any long period of time may be small.
Caa/CCC	Are of poor standing. Such issues may be in default or there may be present elements of danger with respect to principal or interest.
Ca/CC	Are speculative in a high degree. Such issues are often in default or have other marked shortcomings.
C/C, D	Lowest-rated class of bonds, and issues so rated can be regarded as having extremely poor prospects of ever attaining any real investment standing.

*Ratings: Moody's/Standard & Poor's
Source: www.moodys.com

Appendix 2: Portfolio Analysis & The CAPM

- This material is not mandatory for evaluation in the MiM's FMM.
- Only for students who want to see in a bit more detail how portfolio theory developed, as well as the CAPM.
- And for future reference. Of course. Curiosity is part of life!

Modern Investment Theory

- *Investors should control the risk of their portfolio* not by re-allocating among risky assets, but *through the split between risky assets and the risk-free asset.*
- *The optimal portfolio of risky assets* should contain a large number of assets – it *should be well diversified – and is the same for all investors.*

An Individual Security: Return and Risk

- Characteristics of individual securities that are of interest are the:
  - Expected Return;
  - Variance and Standard Deviation of the return;
  - Covariance or Correlation (to another security or index).

Expected Return (or Mean Return)

$$\boxed{E[r] = \sum_{s=1}^{S} p_s r_s}$$

where

r_s :return if state (scenario) s occurs

p_s : probability of state s happening

Example: Consider the following two risky asset world.

- There is a 1/3 chance of each state of the economy, and the only assets are stock A and stock B

		Rate of Return	
Scenario	***Probability***	***Stock A***	***Stock B***
Recession	33,3%	-8%	5%
Normal	33,3%	12%	25%
Boom	33,3%	30%	-5%

$$E(r_A) = \frac{1}{3} \times (-8\%) + \frac{1}{3} \times (12\%) + \frac{1}{3} \times (30\%) = 11.33\%$$

Example: Expected Return

Scenario	Stock A Rate of Return	Stock A Squared Deviation	Stock B Rate of Return	Stock B Squared Deviation
Recession	-8%	0,0374	5%	0,0011
Normal	12%	0,0000	25%	0,0278
Boom	30%	0,0348	-5%	0,0178
Expected return	11,33%		8,33%	

$$E(r_B) = \frac{1}{3} \times (5\%) + \frac{1}{3} \times (25\%) + \frac{1}{3} \times (-5\%) = 8.33\%$$

Note: In practice, many times, instead of really forecasting future scenarios and stock returns in each scenario, investors simply use the historical average return (remember previous slides 11).

Variance and Standard Deviation

- The Variance of the Returns measures the deviations towards the mean

$$\boxed{\mathrm{Var}[r] = \sigma^2 = \sum_{s=1}^{S} p_s \times (r_s - \mathrm{E}[r])^2}$$

- A Shortcut

$$\sigma^2 = \sum_{s=1}^{S} p_s r_s^{\ 2} - \mathrm{E}[r]^2$$

- The Standard Deviation is the square root of the variance. Also known as Volatility.

$$\boxed{\sigma = \sqrt{\mathrm{Var}[r]}}$$

Example: Variance of Returns

Scenario	Stock A Rate of Return	Stock A Squared Deviation	Stock B Rate of Return	Stock B Squared Deviation
Recession	-8%	0,0374	5%	0,0011
Normal	12%	0,0000	25%	0,0278
Boom	30%	0,0348	-5%	0,0178
Expected return	11,33%		8,33%	
Variance	0,0241		0,0156	

$$\sigma_A^2 = \frac{1}{3}\times(-0.08-0.1133)^2 + \frac{1}{3}\times(0.12-0.1133)^2 + \frac{1}{3}\times(0.3-0.1133)^2$$

$$= \frac{1}{3}\times 0.0374 + \frac{1}{3}\times 0.0000 + \frac{1}{3}\times 0.0348 = 0.0241$$

Example: Standard Deviation

	Stock A		Stock B	
	Rate of	Squared	Rate of	Squared
Scenario	***Return***	***Deviation***	***Return***	***Deviation***
Recession	-8%	0,0374	5%	0,0011
Normal	12%	0,0000	25%	0,0278
Boom	30%	0,0348	-5%	0,0178
Expected return	11,33%		8,33%	
Variance	0,0241		0,0156	
Standard Deviation	15,52%		12,47%	

$$\sigma_A = \sqrt{0.0241} = 0.155 = 15.52\%$$

Covariance and Correlation of Returns

- The Covariance is a measure of simultaneous movement of two stocks:

$$\mathrm{Cov}[r_A, r_B] = \sum_{s=1}^{S} p_s \times (r_{A,s} - \mathrm{E}[r_A])(r_{B,s} - \mathrm{E}[r_B])$$

$$= \sum_{s=1}^{S} p_s \times r_{A,s} \times r_{B,s} - \mathrm{E}[r_A] \times \mathrm{E}[r_B]$$

- The Correlation Coefficient between the returns of two stocks:

$$\rho_{AB} = \frac{\mathrm{Cov}[r_A, r_B]}{\sigma_A \times \sigma_B}$$

Example: Covariance and Correlation

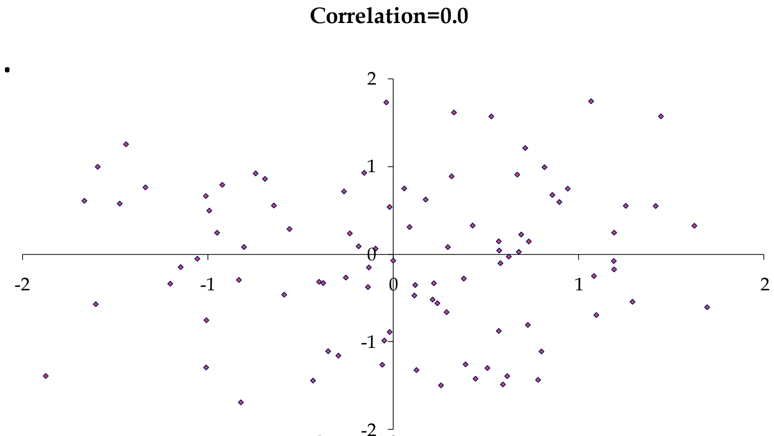
Scenario	Stock A Rate of Return	Stock A Deviation	Stock B Rate of Return	Stock B Deviation
Recession	-8%	-0,1933	5%	-0,0333
Normal	12%	0,0067	25%	0,1667
Boom	30%	0,1867	-5%	-0,1333
Expected return	11,33%		8,33%	

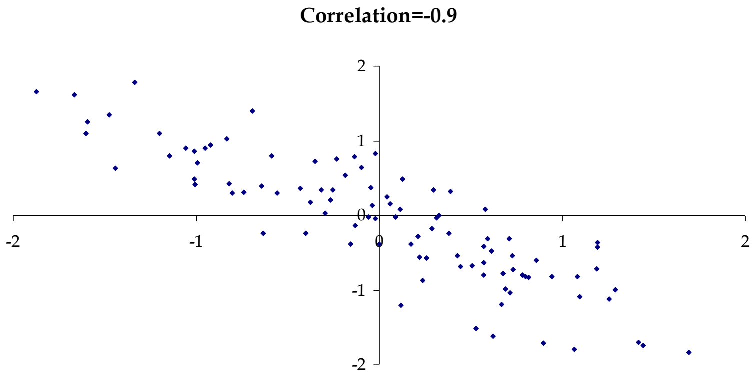
$$Cov(r_A, r_B) = \sigma_{A,B} = \frac{1}{3}\times[(-0.1933)\times(-0.0333)] + \frac{1}{3}\times[0.0067\times 0.1667] + \frac{1}{3}\times[0.1867\times(-0.1333)]$$

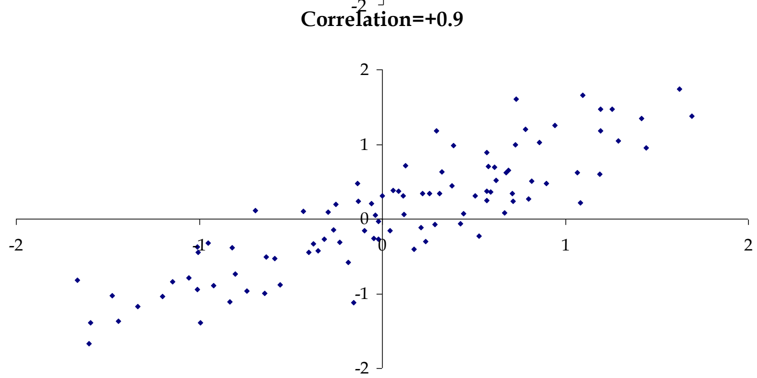
$$= \frac{1}{3}\times 0.0064 + \frac{1}{3}\times 0.00111 + \frac{1}{3}\times(-0.02488) = -0.0058$$

Correlation Coefficient

- The Correlation Coefficient measures the co-movement of two stocks. It varies between -1 and +1.
  - Perfect positive correlation: 1
  - Independence: 0
  - Perfect negative correlation: -1







Estimating Means and Covariances (in practice…)

- In real life we do not know probability of each state of the world and the return that corresponds to it
- We need to use historical data to estimate average returns, variance and covariance of returns

$$\boxed{\bar{r} = \frac{1}{T}\sum_{t=1}^{T} r_t}$$

$$\boxed{\hat{\sigma}^2 = \frac{1}{T-1}\sum_{t=1}^{T}(r_t - \bar{r})^2}$$

$$\boxed{\hat{\sigma}_{ij} = \frac{1}{T-1}\sum_{t=1}^{T}(r_{it} - \bar{r}_i)(r_{jt} - \bar{r}_j)}$$

Estimating Means and Covariances (cont.)

- We can use the functions *average*(), *var*(), and *stdev*() in Excel
- We are implicitly assuming that the returns came from the same probability distribution in each year of the sample
- The estimated mean and variance are themselves random variables since there is estimation error that depends on the particular sample of data used (sampling error)
  - We can calculate the standard error of our estimates and figure out a confidence interval for them
  - This contrasts with the true (but unknown) mean and variance which are fixed numbers, not random variables…
- Annualizing Means and Covariances:
  - Annual return is *approximately* equal to the sum of the 12 monthly returns; assuming monthly returns are independently distributed (a consequence of market efficiency) and have the same variance:
  - If mean, standard deviation or covariance are estimated from historic monthly returns, estimates will be per month.
  - To annualize:
    - mean, variance, covariance: multiply by 12
    - standard deviation: multiply by sqrt(12)

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Forming Portfolios

- Portfolio Weights: fraction of wealth invested in different assets
  - add up to 1.0
  - denoted by ‘w’
- Example
  - $100 MSFT, $200 in GE
    - Total investment: $100+$200=$300
  - Portfolio weights
    - MSFT: $100/$300 = 1/3
    - GE: $200/$300 = 2/3

  - PS: Can we have negative portfolio weights? Yes, if short-selling is allowed (idea is “borrow” the stock now and sell it, and then you’ll have to buy it (pay the price) later).

Portfolio: Expected Return

- Portfolio return (based on realized returns)
  - Average of returns on individual securities weighted by their portfolio weights

$$\boxed{r_P = w_A \times r_A + w_B \times r_B}$$

- The expected return of the portfolio (based on expected returns)

$$\boxed{E(r_P) = w_A \times E(r_A) + w_B \times E(r_B)}$$

Example: Portfolio Expected Return

- Consider a portfolio with 50% weight in Stock A and 50% weight in Stock B:

	Rate of Return			
Scenario	***Stock A***	***Stock B***	***Portfolio***	***squared deviation***
Recession	-8%	5%	-1,5%	0,0128
Normal	12%	25%	18,5%	0,0075
Boom	30%	-5%	12,5%	0,0007
Expected return	11,33%	8,33%	9,8%	

Expected rate of return on the portfolio is a weighted average of the expected returns on stocks in portfolio:

$$E(r_P) = w_A E(r_A) + w_B E(r_B)$$

$$9.8\% = 50\% \times (11.33\%) + 50\% \times (8.33\%)$$

Portfolio: Variance and Standard Deviation

- The Variance of a portfolio is:

$$\boxed{\sigma_P^2 = (w_A\sigma_A)^2 + (w_B\sigma_B)^2 + 2w_A w_B \sigma_{AB}}$$

  - We can use the Correlation term instead of the covariance:

$$\boxed{\sigma_P^2 = (w_A\sigma_A)^2 + (w_B\sigma_B)^2 + 2w_A w_B \sigma_A \sigma_B \rho_{AB}}$$

- The standard deviation of the portfolio is just the square root of the variance of the portfolio.

Example: Portfolio Variance

	Rate of Return			
Scenario	***Stock A***	***Stock B***	***Portfolio***	***squared deviation***
Recession	-8%	5%	-1,5%	0,0128
Normal	12%	25%	18,5%	0,0075
Boom	30%	-5%	12,5%	0,0007
Expected return	11,33%	8,33%	9,8%	
Variance	0,0241	0,0156	0,0070	
Standard Deviation	15,52%	12,47%	8,38%	

Variance of the rate of return on the two-stock portfolio Wa=Wb=1/2:

$$\sigma_P^2 = (w_A\sigma_A)^2 + (w_B\sigma_B)^2 + 2(w_A\sigma_A)(w_B\sigma_B)\rho_{AB}$$

$$= (0.5\times 0.155)^2 + (0.5\times 0.1247)^2 + 2\times 0.5\times 0.155\times 0.5\times 0.1247\times(-0.2985)$$

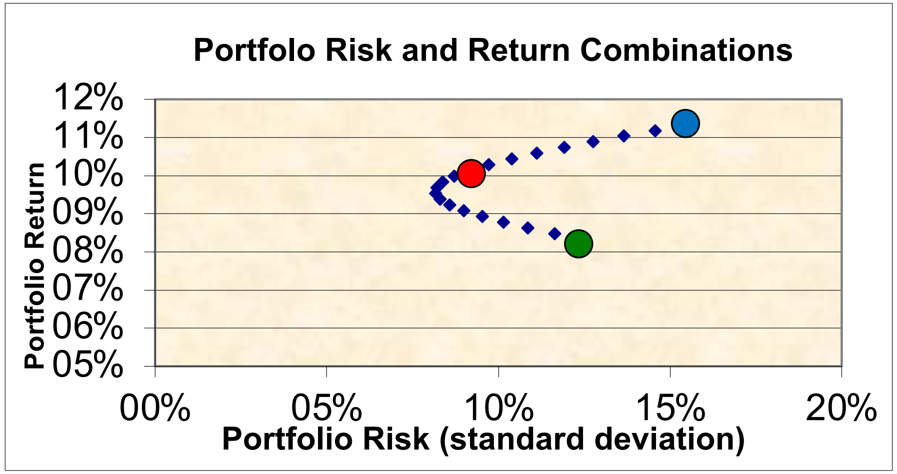
$$= 0.007$$

NOTE: Notice the decrease in risk that diversification offers

Portfolio with 50% in stock A and 50% in stock B has less risk than either stock in isolation!!!

Efficient Frontier: with 2 stocks

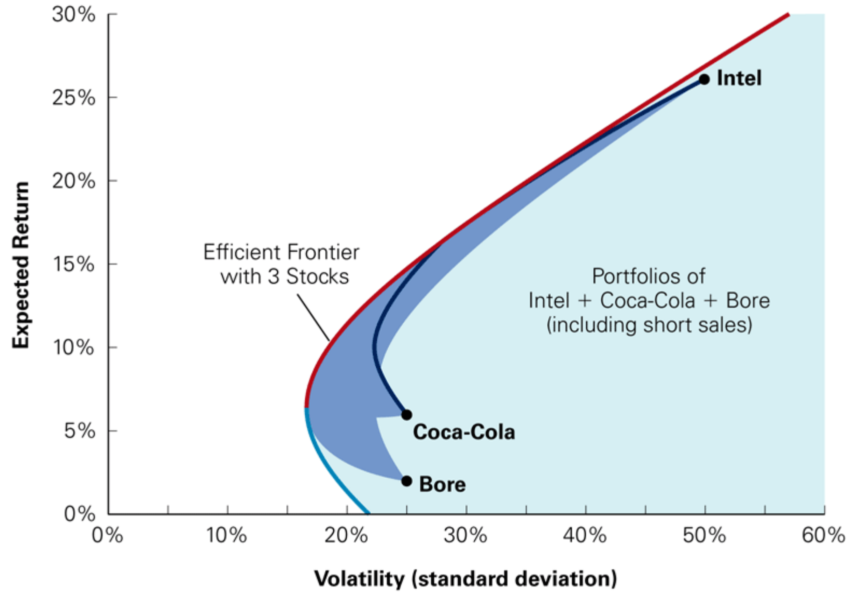
% Stock A	Risk	Return
0%	12,5%	8,3%
5%	11,6%	8,5%
10%	10,9%	8,6%
15%	10,2%	8,8%
20%	9,5%	8,9%
25%	9,0%	9,1%
30%	8,6%	9,2%
35%	8,3%	9,4%
40%	8,2%	9,5%
45%	8,2%	9,7%
50%	8,4%	9,8%
55%	8,7%	10,0%
60%	9,2%	10,1%
65%	9,7%	10,3%
70%	10,4%	10,4%
75%	11,1%	10,6%
80%	11,9%	10,7%
85%	12,8%	10,9%
90%	13,6%	11,0%
95%	14,6%	11,2%
100%	15,5%	11,3%



We can consider other portfolio weights besides 50% in stock A and 50% in stock B.

Note: only efficient to invest “above” the Minimum Variance (SD) portfolio

Efficient Frontier with Many Stocks



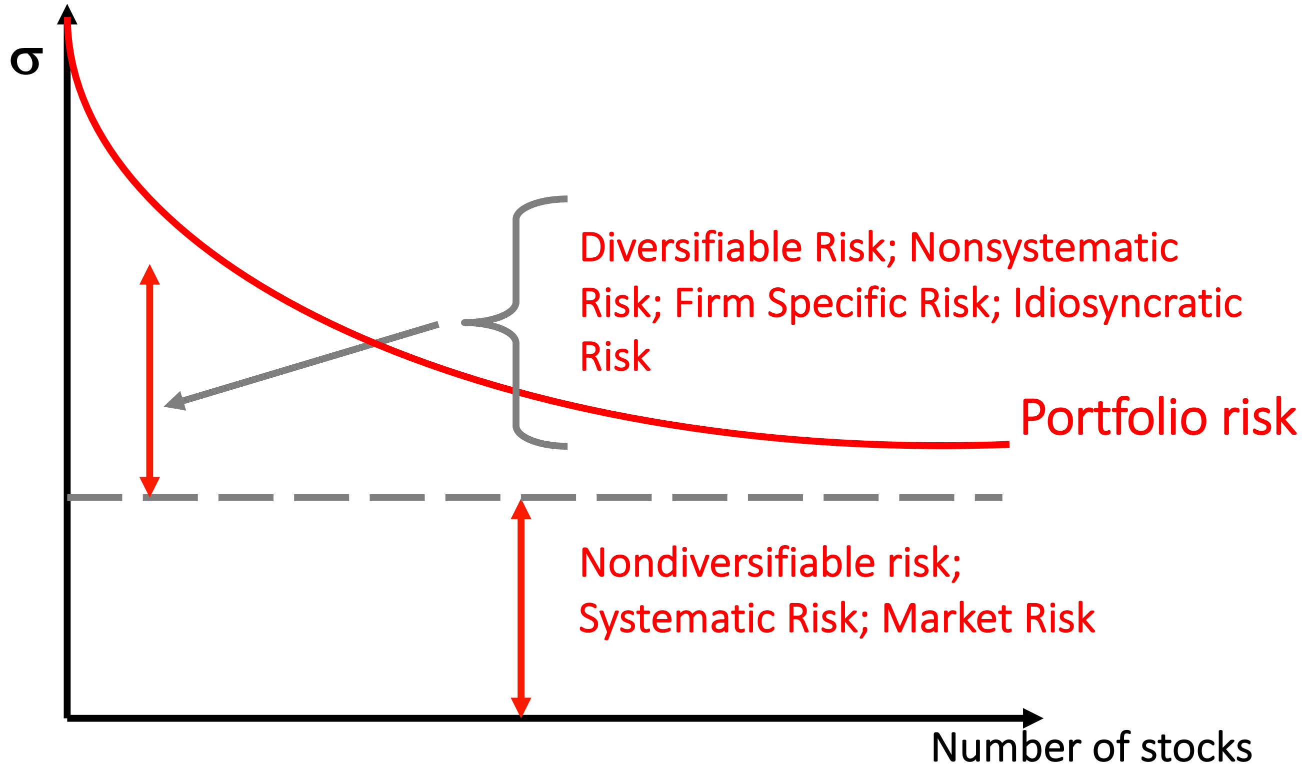
Section of the opportunity set above the minimum variance portfolio is the efficient frontier (north-west edge): offers minimum risk for a given expected return

Diversification Benefit

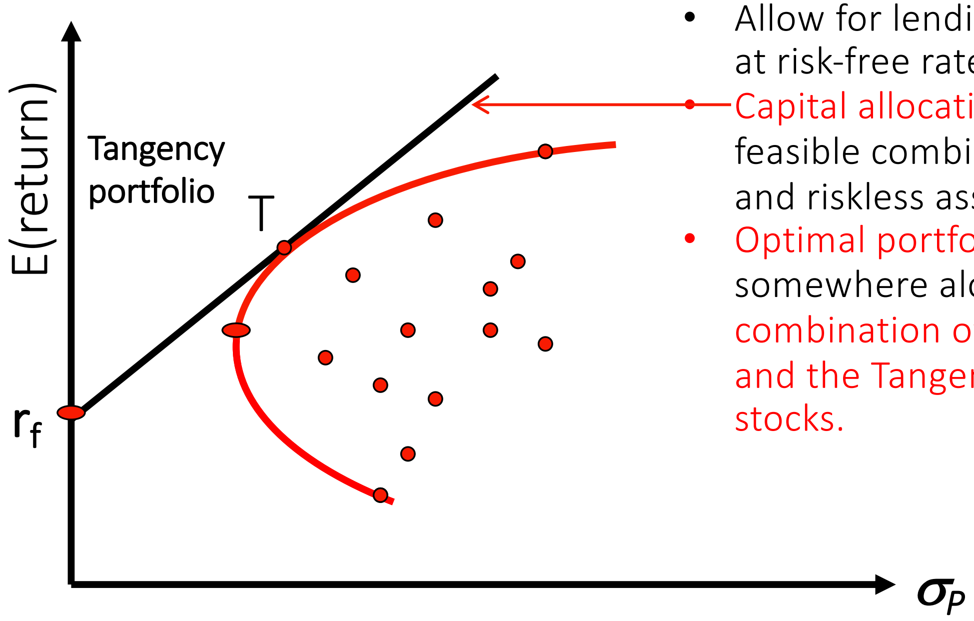
- Diversification can substantially reduce the variability of returns without an equivalent reduction in expected returns.
- This reduction in risk arises because worse than expected returns from one asset are offset by better than expected returns from another
  - However, there is a minimum level of risk that cannot be diversified away, and that is what we call systematic risk.

Portfolio Risk and Limits to Diversification

In a large portfolio the variance terms are effectively diversified away, but the covariance terms are not



Moving to a Pricing Model: Not only (risky) Stocks, but also a Risk-Free Asset



- Allow for lending and borrowing at risk-free rate (T-bill).
- Capital allocation line (CAL): feasible combinations of stock(s) and riskless asset.
- Optimal portfolios are somewhere along the CAL: a combination of the riskfree asset and the Tangency portfolio of stocks.

- If $E(r_P) < E(r_T)$: riskless lending, $w_T < 100\%$
- If $E(r_P) > E(r_T)$: riskless borrowing, $w_T > 100\%$